**1.16**

a) The objective function is the number of new SUVs sold, which we can denote by *Q*(*F, G*).

b) The constraint is that total spending must be less than or equal to $2million, or *TS * $2 million.

c) The constrained optimization problem is



d) The following table shows all possible combinations of spending on football games and golf events:

|  |  |  |  |
| --- | --- | --- | --- |
| (*F, G*) | New sales from *F* | New sales from *G* | Total new sales |
| (0, 2) | 0 | 9 | 9 |
| (0.5, 1.5) | 10 | 8 | 18 |
| (1, 1) | 15 | 6 | 21 |
| (1.5, 0.5) | 19 | 8 | 27 |
| (2, 0) | 20 | 0 | 20 |

The table indicates that new SUV sales are maximized when (*F, G*) = (1.5, 0.5), that is, when the manufacturer spends $1.5 million on football and $0.5 million on golf.

**2.6.**

The demand for apple pies is *Qd* = 54 – 6*P*.



To find the equation of the demand curve, observe that when she drops the price by $0.50, she sells 3 more pies. So, movement along the demand occurs so that



The demand curve then has the form *Qd = A – 6P*, where *A* is a constant. We can determine the value of *A* using any one of the three data points on the demand curve. For example, if we use the point *P* = 5 and *Q* = 24, we see that 24 = *A* – 6(5), so that *A* = 54. So the demand curve can be described by the equation *Qd* = 54 – 6*P*.

To find elasticity of demand at any point on the demand curve, we use formula



**2.22.**

We know that along a linear demand curve



Using the given information this implies



Plugging this result into a demand equation using the known price and quantity then implies



So a demand equation that fits this information is given by



Graphically, the demand curve looks like



**2.27.**

First, consider each demand curve in its “inverse” form: long run demand is *P =* 15 – 0.5*Q*, and short run demand is *P =* 30 – 2*Q*. Thus, the slope of the long run demand is –0.5, which is closer to zero than that of the short run demand, –2. Thus, long run demand is flatter. Second, consider the graph below:



Again, long run demand is flatter and thus more sensitive to changes in price. Consider, for instance a price of $10. Quantity demanded is equal in both the long and short runs at *P =* 10. However, consider increasing the price to, say, $15. Although this will reduce quantity demanded in the short run by a little, it would reduce quantity demanded all the way to zero in the long run.

**2.29**

The equilibrium price in January is equal to *P* = 3 and equilibrium quantity is equal to *Q* = 60. We find equilibrium price by solving *Qs* = *Qd*, which is 30∙*P* – 30 = 120 – 20∙*P*. When we have equilibrium price we can substitute it to either the demand function or supply function, since they have to give the same quantity at that price, and obtain equilibrium quantity equal to *Q* = 60. After the supply decreases in February, new equilibrium price is per mile is equal to *P* = $3.60, while the demanded quantity is equal to *Q* = 48. When the demand goes up in March, the quantity in equilibrium is the same as in January but price is even higher and equal to *P* = $4. All those changes are illustrated on the graph below.



**3.8**

This utility function does have the property of diminishing *MRSx*,y. One way to verify this is to graph several indifference curves. Another way to tell is to use algebra. Recall that  Applying that general formula to this case gives us  As we move “down” the indifference curve, *x* increases and *y* decreases. As *y* decreases,  decreases. Thus, *MRSx,y* decreases.

**3.13**

In the following pictures, *U*2 > *U*1.

a)



b)



c)



d)



**3.17 Answer all parts of Problem 3.15 for the utility function *U*(*x*, *y*) = *xy* + *x*. The marginal utilities are *MUx* = *y* + 1 and *MUy* = *x*.**

a) Yes, the “more is better” assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of  remains constant as the consumer buys more *x*.

c) 

d) As the consumer substitutes  for , the  will diminish.

e & f) See figure below. The indifference curves intersect the *x*-axis, since it is possible that *U >* 0 even if *y* = 0.

**3.21**

a) Yes, the “more is better” assumption is satisfied for both goods since both marginal utilities are always positive.

b) Since we do not know the value of , only that it is positive, we need to specify three possible cases:

When , the marginal utility of  diminishes as  increases.

When , the marginal utility of  remains constant as  increases.

When , the marginal utility of  increases as  increases.

c) 

d) As the consumer substitutes  for , the  will diminish.

e & f) The graph below depicts indifference curves for the case where  and  Thus . Regardless, the indifference curves will never intersect either axis.

**3.22**

a) Yes, the “more is better” assumption is satisfied for both goods since both marginal utilities are always positive.

b) The marginal utility of  increases as the consumer buys more .

c) 

d) As the consumer substitutes  for , the  will diminish.

e) Since it is possible to have *U >* 0 if either *x =* 0 (and *y >* 0) or *y =* 0 (and *x* > 0), the indifference curves intersect both axes.

f) The slope of a typical indifference curve at some basket  is the . At , . Note that this holds regardless of the value of . Therefore, the slope of any indifference curve at  will be .

**4.5**

This question cannot be solved using the usual tangency condition. However, you can see from the graph below that the optimum basket will necessarily lie on the “elbow” of some indifference curve, such as (5, 3), (10, 6) etc. If the consumer were at some other point, he could always move to such a point, keeping utility constant and decreasing his expenditure. The equation of all these “elbow” points is 3*x* = 5*y*, or *y =* 0.6*x*. Therefore the optimum point must be such that 3*x* = 5*y.*

The usual budget constraint must hold of course. That is, . Combining these two conditions, we get (*x*, *y*) = (20, 12).

(5,3)

(10,6)

*y*

*x*

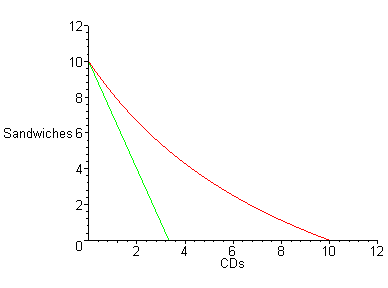
(20,12)

**4.9**

See the graph below. The fact that Helen’s indifference curves touch the axes should immediately make you want to check for a corner point solution.

To see the corner point optimum algebraically, notice if there was an interior solution, the tangency condition implies (*S* + 10)/(*C* +10) = 3, or *S* = 3*C* + 20. Combining this with the budget constraint, 9*C +* 3*S =* 30, we find that the optimal number of CDs would be given by  which implies a negative number of CDs. Since it’s impossible to purchase a negative amount of something, our assumption that there was an interior solution must be false. Instead, the optimum will consist of *C =* 0 and Helen spending all her income on sandwiches: *S =* 10.

Graphically, the corner optimum is reflected in the fact that the slope of the budget line is steeper than that of the indifference curve, even when *C =* 0. Specifically, note that at (*C*, *S*) = (0, 10) we have *PC* / *PS* = 3 > *MRSC,S* = 2. Thus, even at the corner point, the marginal utility per dollar spent on CDs is lower than on sandwiches. However, since she is already at a corner point with *C =* 0, she cannot give up any more CDs. Therefore the best Helen can do is to spend all her income on sandwiches: (*C*, *S*) = (0, 10). [Note: At the other corner with *S* = 0 and *C =* 3.3, *PC* / *PS* = 3 > *MRSC,S* = 0.75. Thus, Helen would prefer to buy more sandwiches and less CDs, which is of course entirely feasible at this corner point. Thus the *S =* 0 corner cannot be an optimum.]



**4.15**

When Justin maximizes utility, his optimal consumption basket will be on the budget constraint and satisfy the tangency condition.  
Any basket on the budget line will satisfy pxx + pyy = I, or 2x + 5py = 40.  
The tangency condition requires that MUx / px = MUy / py, or that 5 / 2 = x / py. This implies that 5py = 2x.  
Putting these two equations together reveals that 5py + 5py = 40; thus py = 4.

**4.17**



Assume Joe is initially at an interior optimum, point *A*, and that the price of other goods is $1. Let Joe’s income be *M*. Point *A* then consists of *GA =* 1000 units of root beer and *YA = M –* 2000 units of other goods. The effect of the proposal is to rotate the budget line inward (the price change) and then shift it outward (the cash transfer), for a total movement from *BL*1 to *BL*2. Notice that *BL*2 intersects *BL*1 exactly at point *A*: after the price increase, (*GA*, *YA*) costs Joe 1000\*2.50 + *M* – 2000= *M* + 500, which is equal to his income after the cash transfer.

Because *A* was initially optimal, *MRSG,Y* = 2 at point *A*. Yet the price ratio along *BL*2 is 2.5. Hence *MRSG,Y* < *PG* / *PY*, so Joe can increase his utility by purchasing less gas and more of the composite good, at a point such as *B* depicted in the graph above.Thus, the proposal will make Joe better off.

**5.5**

a) Since she spends 25% of her income on *x*, it must be true that *pxx/I = 0.25­.*  Thus *x/I = 0.25­/px*. This means that *x/I* is a constant. If *I* increases by 1%, *x* must also increase by 1%. Since the percentage increase in *x* is the same as the percentage increase in *I*, the income elasticity must be 1.

b) The income elasticity of demand would still be 1. Now *x/I = 0.6­/px*. This means that *x/I* is a constant. If *I* increases by 1%, *x* must also increase by 1%.

**5.7**

a) Karl’s optimal bundle will always be such that 2*H* = 3*B*. If this were not true then he could decrease the consumption of one of the two goods, staying at the same level of utility and reducing expenditure. Also, at the optimal bundle, it must be true that . Substituting the first condition into the second we get  which implies that the demand curve for beer is given by,

b) You can answer this just by looking at the demand curve. Because it has a larger coefficient, the price of hamburgers affects the demand for beer more than the price of beer. A one dollar increase in  decreases demand for beer more than a one dollar increase in .

**5.11**

a) The budget constraint is  and the tangency condition is . Solving, the optimal bundle is (*x*, *y*)=(20, 40) with a utility of 202(40)=16,000.

b) Now *py*=8. We need to calculate *px* such that, with the new prices, Ginger reaches exactly the same indifference curve as before. The new optimal bundle (x,y) must be such that: . The tangency condition now implies that  that is,  Substituting this into the budget constraint we find that y=10. Using the condition , we find that *x* = 40. Finally, substituting the values of x and y back into the budget constraint, we can see that , or *px*=4. Therefore, if the price of *y* were to increase to $8, Ginger would need the price of *x* to decrease to $4 in order to be exactly as well off as before.

**5.16**

a) If we are at an interior optimum the tangency condition must hold:



Substituting into the budget line, , yields



b) If , then



Since we must have , we must have



So the consumer would only purchase  for prices less than 10.

c)

Given , the slope of the budget line is –1. At the corner point optimum, the slope of the indifference curve is



Because the indifference curve has a flatter slope than the budget line, the consumer would like to substitute more  for , but has no more  to give up at the corner point.

d) . If the consumer were to purchase any , since the “bang for the buck” for  is less than the “bang for the buck” for , the consumer would reduce total utility by increasing  above zero.

e)

As shown in part a), the demand for  depends only on  and . Therefore, the location of the demand curve does not depend on .

**5.20 .**

a) Using the tangency condition, , and the budget constraint, , Lou’s initial optimum is the basket (*x*, *y*) = (15, 60) with a utility of 900.

b) First we need the decomposition basket. This would satisfy the new tangency condition,  and would give him as much utility as before, i.e. . This gives  or approximately (17.3,51.9). Now we need the final basket, which satisfies the same tangency condition as the decomposition basket and also the new budget constraint:  Together, these conditions imply that (*x*, *y*) = (20, 60). The substitution effect is therefore 17.3 – 15 = 2.3, and the income effect is 20 – 17.3 = 2.7.

**5.29**

If Terry’s wage rate is *w*, then the income he earns from working is (24 – *L*)*w*. Since *PY* = 1, the number of units of other goods he purchases is *Y* = (24 – *L*)*w*.

Now at an optimal bundle, Terry’s  must equal the price ratio *w/PY = w.* Therefore, the tangency condition tells us that . The two conditions imply . This means that the optimal amount of leisure is *L* = 11.5. You can see that this does not depend on the wage rate.